Sedra/Smith
Microelectronic Circuits 6/E

Chapter 4-B
Bipolar Junction Transistors (BJTs)
4.5 Small-Signal Operation And Models

\[ I_C = I_s e^{V_{BE}/V_T} \]
\[ I_B = I_C / \beta \]
\[ I_E = I_C / \alpha \]
\[ V_C = V_{CE} = V_{CC} - I_C R_C \]
4.5.1 The collector Current and the Transconductance

\[ V_{BE} = V_{BE} + V_{be} \]

\[ i_C = I_s e^{V_{BE}/V_T} = I_s e^{(V_{BE} + V_{be})/V_T} = I_s e^{(V_{BE}/V_T)} \cdot e^{(V_{be}/V_T)} \]

\[ = I_C e^{(V_{be}/V_T)} = I_C \left(1 + \frac{V_{be}}{V_T}\right) = I_C + I_C \frac{V_{be}}{V_T} \]

\[ i_c = g_m V_{be} = I_C \frac{V_{be}}{V_T} \Rightarrow g_m = \frac{I_C}{V_T} \quad (4.45) \]
The graph shows a plot of $i_C$ versus $v_{BE}$ with $v_{be}$ and $V_{BE}$ indicated. The slope at point $Q$ is labeled as $g_m$. The equation given is:

$$g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{i_c=I_C}$$

(4.46)
4.5.2 The Base Current and the Input Resistance at the Base

\[ i_B = \frac{i_C}{\beta} = \frac{I_C + I_C \frac{v_{be}}{V_T}}{\beta} = \frac{I_C}{\beta I_B} + \frac{1}{\beta} \frac{I_C}{i_b} v_{be} \]

\[ i_b = \frac{1}{\beta \frac{I_C}{V_T}} v_{be} = \frac{g_m}{\beta} v_{be} \]

\[ r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} \quad (4.51) \]

\[ r_\pi = \frac{\beta}{\frac{I_C}{V_T} = \frac{V_T}{I_B}} \quad (4.52) \]
4.5.3 The Emitter Current and the Input Resistance at the Emitter

\[ i_E = \frac{i_C}{\alpha} = \frac{I_C + i_c}{\alpha} = \frac{I_C}{I_E} + \frac{i_c}{i_e} \]

\[ i_e = \frac{i_c}{\alpha} = \frac{1}{\alpha} \frac{I_C}{V_T} \nu_{be} = \frac{g_m}{\alpha} \nu_{be} \quad (4.54) \]

\[ r_e \equiv \frac{\nu_{be}}{i_e} \Rightarrow r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} \quad (4.57) \]

\[ \Rightarrow \frac{\alpha}{I_C} = \frac{V_T}{I_E} \quad (4.56) \]
The relationship between $r_\pi$ and $r_e$ can be found as

$$v_{be} = i_b r_\pi = i_e r_e$$

thus

$$r_\pi = \frac{i_e}{i_b} r_e = (1 + \beta) r_e$$

(4.58)

Figure 4.38 Illustrating the definition of $r_\pi$ and $r_e$
4.5.4 Voltage Gain

\[ v_C = V_{CC} - i_C R_C \]

\[ = V_{CC} -(I_C + i_c) R_C \]

\[ = (V_{CC} - I_C R_C ) - i_c R_C \]

\[ = V_C - i_c R_C \]

\[ v_{ce} = -i_c R_C = -g_m v_{be} R_C \]

\[ = (-g_m R_C) v_{be} \]

\[ A_v \equiv \frac{v_c}{v_{be}} = -g_m R_C = -\frac{I_C}{V_T} R_C \]
4.5.5 Separating the Signal and the DC Quantities

Figure 4.39  The amplifier circuit of Fig. 4.36(a) with the dc sources ($V_{BE}$ and $V_{CC}$) eliminated (short circuited). Thus only the signal components are present. Note that this is a representation of the signal operation of the BJT and not an actual amplifier circuit.
4.5.6 The Hybrid- Model

\[ i_b = \frac{v_{be}}{r_\pi} \]

\[ i_c = g_m v_{be} \]

\[ i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} \]

\[ = \frac{v_{be}}{r_\pi} \left(1 + g_m r_\pi \right) \]

\[ = \frac{v_{be}}{r_\pi} \left(1 + \beta \right) = \frac{v_{be}}{r_e} \]

\[ g_m v_{be} = g_m \left(i_b r_\pi \right) = \left(g_m r_\pi \right) i_b = \beta i_b \]
4.5.7 The T Model

\[ i_b = \frac{v_{be}}{r_e} - g_m v_{be} \]

\[ = \frac{v_{be}}{r_e} \left( 1 - g_m r_e \right) \]

\[ = \frac{v_{be}}{r_e} (1 - \alpha) \]

\[ = \frac{v_{be}}{r_e} \left( 1 - \frac{\beta}{1 + \beta} \right) \]

\[ = \frac{v_{be}}{r_e \left( 1 + \beta \right)} = \frac{v_{be}}{r_{\pi}} \]

\[ g_m v_{be} = g_m (i_e r_e) = (g_m r_e) i_e = \alpha i_e \]
4.5.9 Application of the Small-Signal Equivalent Circuits

Analysis process:

1. Determine the dc operating point of the BJT and in particular the dc collector current $I_C$.

2. Calculate the values of the small-signal model parameters: $g_m, r_\pi, r_e$.

3. Eliminate the dc sources by replacing each dc voltage source with a short circuit and each dc current source with an open circuit.

4. Replace the BJT with one of its small-signal equivalent circuit models.

5. Analyze the resulting circuit to determine the required quantities.
   (e.g., voltage gain, input/output resistance)
Example 4.14  We wish to analyze the transistor amplifier shown in Fig.4.42(a) to determine its voltage gain. Assume $\beta = 100$.
Determine the dc operating point of the BJT and in particular the dc collector current $I_C$

Step 1:

we assume $v_i = 0$

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}} \approx \frac{3 - 0.7}{100} = 0.023\text{mA}$$

$$I_C = \beta I_B = 100 \times 0.023\text{mA} = 2.3\text{mA}$$

$$V_C = V_{CC} - I_C R_C = 10 - 2.3 \times 3 = 3.1\text{V}$$
Step 2:

Calculate the values of the small-signal model parameters: $g_m, r_\pi, r_e$.

$$r_e = \frac{V_T}{I_E} = \frac{25\text{mA}}{2.323\text{mA}} = 10.8\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{2.3\text{mA}}{25\text{mA}} = 92\text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{92\text{mA/V}} = 1.09\text{k}\Omega$$
Step 3 and 4:
Eliminate the dc sources by replacing each dc voltage source with a short circuit and each dc current source with an open circuit. Replace the BJT with one of its small-signal equivalent circuit models.
step 5:
Analyze the resulting circuit to determine the required quantities.

\[ v_{be} = v_i \frac{r_\pi}{r_\pi + R_{BB}} = \frac{1.09}{101.09} v_i = 0.011 v_i \]

\[ v_o = -g_m v_{be} R_C = -92 \times 0.011 v_i \times 3 = -3.04 v_i \]

\[ A_v = \frac{v_o}{v_i} = -3.04 \] (The "-" sign indicated a phase reversal)
Example 4.15  We wish to consider the waveforms at various point in the circuit analyzed in the example 4.14

If we take the triangular waveform $v_{be(p-p)}$ to be 20mV

$$\hat{V}_i = \frac{\hat{V}_{be}}{0.011} = \frac{10}{0.011} = 0.91V$$

We will use a somewhat lower value for $\hat{V}_i$ of approximately 0.8V
\[ \hat{I}_b = \frac{\hat{V}_i}{R_{BB} + r_\pi} = \frac{0.8}{100 + 1.09} = 0.008 \text{mA} \]

\[ i_b = \frac{v_{be}}{r_\pi} = (0.8 \times 0.011)/1.09\text{k}\Omega = 0.00081 \text{mA} \]
\[ \hat{V}_{be} = \hat{V}_i \frac{r_\pi}{R_{BB} + r_\pi} = 0.8 \frac{1.09}{100 + 1.09} = 8.6 \text{mV} \]
\[ \hat{I}_c = \beta \hat{i}_b = 100 \times 0.008 = 0.8\text{mA} \]
\[ \hat{V}_c = 3.04 \times 0.8 = 2.43 \text{V} \]
Example 4.16 We need to analyze the circuit of Fig.4.44(a) to determine the voltage gain ($\alpha = 0.99$).
**Sol:**

\[ I_E = \frac{10 - V_E}{R_E} \approx 0.93 \text{mA} \]

\[ I_C = 0.99 I_E = 0.92 \text{mA} \]

\[ V_C = -10 + I_C R_C = -10 + 0.92 \times 5 = -5.4 \text{V} \]

The transistor is in the active mode.
Figure 4.44 Example 4.16: (c) small-signal model; (d) small-signal analysis performed directly on the circuit.

\[ g_m = \frac{I_c}{V_T} = 36.8 \text{mA} \]

\[ r_e = \frac{V_T}{I_E} = 27.2 \Omega \]

\[ r_{\pi} = \frac{\beta}{g_m} = 2.72k\Omega \]

\[ i_e = -\frac{v_i}{r_e}, \quad v_o = -\frac{\alpha R_C}{r_e} v_i \]

\[ A_v = \frac{v_o}{v_i} = -\frac{\alpha R_C}{r_e} \]

\[ = -\frac{0.99 \times 5k\Omega}{27\Omega} = -183.3V/V \]
Performing Small-Signal Analysis Directly on the Circuit Diagram

\[\frac{v_i}{R_{BB} + r_{\pi}} i_b \]

\[i_C = \beta i_b\]

\[v_o = -R_C i_C = -\beta i_b R_C\]

\[A_v = \frac{v_o}{v_i} = -\frac{\beta R_C}{R_{BB} + r_{\pi}}\]

Figure 4.46 Performing signal analysis directly on the circuit diagram with the BJT small-signal model implicitly employed circuit for example 4.14
4.5.11 Augmenting the Small-Signal Model to Account for the Early Effect

**Figure 4.47** The hybrid-\(\pi\) small-signal model, in its two versions, with the resistance \(r_o\) included.
\[ v_o = -g_m v_{be} \left( R_C \parallel r_o \right) \]

Thus the gain will be somewhat reduced. Obviously if \( r_o >> R_C \), the reduction in gain will be negligible, and one can ignore the effect of \( r_o \). In general, in such a configuration \( r_o \) can be neglected if it is greater than \( 10R_C \).
4.5.12 Summary

Hybrid- Model

T- Model
4.6 Basic BJT Amplifier Configurations

4.6.1 The Three Basic Configurations

(a) Common-Emitter (CE)

(b) Common-Base (CB)

(c) Common-Collector (CC) or Emitter Follower
4.6.2 Characterizing Amplifiers

\[ v_i = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} v_{\text{sig}}, \quad v_o = \frac{R_L}{R_o + R_L} A_{v_o} v_i \]
Input resistance: \( R_{\text{in}} \equiv \frac{v_i}{i_i} \)

Open-circuit voltage gain: \( A_{vo} \equiv \left. \frac{v_o}{v_i} \right|_{R_L=\infty} \)

Voltage gain with \( R_L \): \( A_v \equiv \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o} \)

Short-circuit current gain: \( A_{is} \equiv \left. \frac{i_o}{i_i} \right|_{R_L=0} \)

Current gain: \( A_i \equiv \frac{i_o}{i_i} \)
Output resistance: \( R_o \equiv \frac{v_x}{i_x} \bigg|_{v_i=0, R_L=\infty} \)

Overall voltage gain: \( G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \cdot \frac{v_o}{v_i} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_v \)
4.6.3 The Common-Emitter (CE) Amplifier

Characterizing Parameters of CE Amplifier

\[ R_{\text{in}} = r_{\pi} \quad (4.69) \]
\[ v_o = -(g_m v_{\pi})(R_C \parallel r_o) \]
\[ A_{vo} = -g_m \left( R_C \parallel r_o \right) \approx -g_m R_C \quad (4.71) \]
\[ R_o = R_C \parallel r_o \approx R_C \quad (4.73) \]
Concludes:

1. The \( R_{\text{in}} = r_\pi = \beta/g_m = \beta V_T/I_C \) is moderate to low in value (typically, in the kilohm range). Obviously \( R_{\text{in}} \) is directly dependent on \( \beta \) and is inversely proportional to the \( I_C \).

2. The \( R_\circ \approx R_C \) is moderate to high in value (typically, in the kilohm range). Reducing \( R_C \) to lower \( R_\circ \) is usually not a viable proportional because the \( A_v \) is also reduced.

3. The open circuit voltage gain \( A_{v_0} \) can be high, making the CE configuration the work-hourse in BJT amplifier design. However the bandwidth of the CE amplifier is severely limited.
Overall Voltage Gain

\[ A_v = \frac{v_o}{v_i} = -g_m \left( R_C \parallel R_L \parallel r_o \right) \quad (4.75) \]

\[ v_i = v_{\text{sig}} \frac{r_\pi}{r_\pi + R_{\text{sig}}} \quad (4.74) \]

\[ G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \cdot \frac{v_o}{v_i} = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m \left( R_C \parallel R_L \parallel r_o \right) \quad (4.76) \]

Alternative Gain Expressions

\[ A_v = -\alpha \left( \frac{R_C \parallel R_L \parallel r_o}{r_e} \right) = -\alpha \left( \frac{\text{Total resistance in collector}}{\text{Total resistance in emitter}} \right) \quad (4.78) \]

\[ G_v = -\frac{g_m r_\pi}{r_\pi + R_{\text{sig}}} \left( R_C \parallel R_L \parallel r_o \right) = -\beta \left( \frac{R_C \parallel R_L \parallel r_o}{r_\pi + R_{\text{sig}}} \right) \]

\[ = -\beta \left( \frac{\text{Total resistance in collector}}{\text{Total resistance in base}} \right) \quad (4.80) \]
Performing the Analysis Directly on the Circuit

\[ g_m v_\pi = \beta i_b \]

\[ v_i = v_\pi \]

\[ R_{in} = r_\pi \]

\[ v_o = -g_m v_\pi \left( R_C \parallel r_o \right) \]

\[ R_o = R_C \parallel r_o \]
Example 4.17  A CE amplifier utilized a BJT with $\beta = 100$ and $V_A = 100\text{V}$, is biased $I_C = 1\text{mA}$ and has a $R_C = 1\text{k}\Omega$. Find $R_{\text{in}}, R_{o}, A_{vo}$. If $R_{\text{sig}} = 5\text{k}\Omega, R_L = 5\text{k}\Omega$, Find $G_v, A_v$. If $\hat{v}_\pi$ is to be limited to 5mV, what are the corresponding $\hat{v}_{\text{sig}}$ and $\hat{v}_o$ with the load connected?

Sol: At $I_C = 1\text{mA}$

$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 40\text{mA/V}, \quad r_\pi = \frac{\beta}{g_m} = \frac{100}{40\text{mA/V}} = 2.5\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{1\text{mA}} = 100\text{k}\Omega$$

The amplifier characteristic can now be found as

$$R_{\text{in}} = r_\pi = 2.5\text{k}\Omega \, \uparrow$$

$$A_{vo} = -g_m \left( R_C // r_o \right) = -40\text{mA/V} \cdot (5\text{k}\Omega // 100\text{k}\Omega) = -190.5\text{V/V} \, \uparrow$$

$$R_o = R_C // r_o = 5 // 100 = 4.76 \text{ k}\Omega \, \uparrow$$
\[ A_v = \frac{V_o}{V_i} = -g_m \left( R_C \parallel R_L \parallel r_o \right) = -40 \left( \frac{5}{5} \parallel 100 \right) = -97.6 \text{V/V} \]

\[ G_v = \frac{r_\pi}{r_\pi + R_{\text{sig}}} A_v = -\frac{2.5}{2.5 + 5} \cdot 97.6 \text{V/V} = -32.5 \text{ V/V} \]

If the maximum amplitude of \( v_\pi \) is to be 5mV, the corresponding value of \( \hat{v}_{\text{sig}} \) will be

\[ \hat{v}_{\text{sig}} = \left( \frac{R_{\text{in}} + R_{\text{sig}}}{R_{\text{in}}} \right) \hat{v}_{\pi} = \frac{2.5 + 5}{2.5} \times 5 = 15 \text{mV} \]

\[ \hat{v}_o = |G_v| \cdot \hat{v}_{\text{sig}} = 32.5 \text{ V/V} \cdot 15 \text{mV} = 0.49 \text{V} \]
4.6.4 The Common-Emitter (CE) Amplifier with an Emitter Resistance
\[ i_b = (1 - \alpha) i_e = \frac{i_e}{1 + \beta}, \quad v_i = i_e (r_e + R_e) \]

\[ R_{in} = \frac{v_i}{i_b} = (1 + \beta)(r_e + R_e) \tag{4.83} \]

\[ \frac{R_{in \text{ (with } R_e \text{ include)}}}{R_{in \text{ (without } R_e \text{ )}}} = \frac{(1 + \beta)(r_e + R_e)}{(1 + \beta)r_e} = 1 + \frac{R_e}{r_e} = 1 + g_m R_e \tag{4.84} \]

\[ v_o = -i_e R_C = -\alpha i_e R_C = -\alpha \frac{v_i}{r_e + R_e} \Rightarrow A_{vo} = -\alpha \left( \frac{R_C}{R_e + r_e} \right) \tag{4.85} \]

\[ A_{vo} = -\alpha \left( \frac{R_C}{R_e + r_e} \right) \frac{v_i}{r_e} = -\alpha \left( \frac{R_C}{r_e \left( 1 + \left( \frac{R_e}{r_e} \right) \right)} \right) = -\frac{g_m R_C}{1 + g_m R_e} \tag{4.86} \]

\[ R_o = R_C \parallel r_o \approx R_C \]
Overall Voltage Gain

\[
A_v = \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o} = -\alpha \frac{R_C}{r_e + R_e} \frac{R_L}{R_L + R_C} = -\alpha \frac{R_C // R_L}{r_e + R_e} \tag{4.87}
\]

\[
G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \cdot \frac{v_o}{v_i} = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \cdot \alpha \frac{R_C // R_L}{r_e + R_e}
\]

\[
= -\frac{(1 + \beta) r_e + R_e}{(1 + \beta)(r_e + R_e) + R_{\text{sig}}} \cdot \frac{\beta}{1 + \beta} \frac{R_C // R_L}{r_e + R_e}
\]

\[
= -\beta \frac{R_C // R_L}{R_{\text{sig}} + (1 + \beta)(r_e + R_e)} \tag{4.88}
\]

\[
\frac{v_\pi}{v_i} = \frac{r_e}{r_e + R_e} = \frac{1}{1 + g_m R_e} \tag{4.89}
\]
Concludes:

1. The input resistance $R_{\text{in}}$ is increased by the factor $(1 + g_m R_e)$.
2. The voltage gain from base to collector, $A_v$ is reduced by the factor $(1 + g_m R_e)$.
3. For the same nonlinear distortion, the input signal $v_i$ can be increased by the factor $(1 + g_m R_e)$.
4. The overall voltage gain is less dependent on the value of $\beta$.
5. The high frequency response is significantly improved.
Example 4.18  For the CE amplifier specified in Example 4.17, with value of $R_e$ is needed to raise $R_{in}$ to a value four times that of $R_{sig}$? With $R_e$ included, find $A_{vo}$, $R_o$, $A_v$, and $G_v$. Also, if $\hat{v}_\pi$ is limited to 5mV, what are the corresponding values of $\hat{v}_{sig}$ and $\hat{v}_o$?

Sol:

To obtain $R_{in} = 4R_{sig} = 20k\Omega$

$20k\Omega = (1 + \beta)(r_e + R_e) \Rightarrow r_e + R_e = 20000\Omega/100 = 200\Omega$

$R_e = 200 - 25 = 175\Omega$

$A_{vo} = -\alpha \frac{R_C}{r_e + R_e} = -25. \quad R_o = R_C = 5k\Omega.$

$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -12.5. \quad G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v = -10 \text{ V/V}.$
For $\hat{v}_\pi = 5\text{mV}$, the corresponding value of $\hat{v}_{\text{sig}}$ will be

\[
\hat{v}_i = \left( \frac{r_e + R_e}{r_e} \right) \hat{v}_\pi = 5 \left( 1 + \frac{175}{25} \right) = 40\text{mV},
\]

\[
\hat{v}_{\text{sig}} = \left( \frac{R_{\text{in}} + R_{\text{sig}}}{R_{\text{in}}} \right) \hat{v}_i = 40\text{mV} \times \left( 1 + \frac{5}{20} \right) = 50\text{mV}
\]

\[
\hat{v}_o = |G_v| \cdot \hat{v}_{\text{sig}} = 10\text{V/V} \cdot 50\text{mV} = 0.5\text{V}
\]
4.6.5 The Common-Base (CB) Amplifier

\[ R_{in} = r_e \quad (4.90) \]
\[ v_o = -\alpha i_e R_C, \quad v_i = -i_e r_e \]
\[ A_{vo} \equiv \frac{v_o}{v_i} = \frac{\alpha}{r_e} R_C = g_m R_C \quad (4.71) \]
\[ R_o = R_C \quad (4.73) \]
Overall Voltage Gain

\[
\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{r_e}{r_e + R_{\text{sig}}}
\]

\[
A_v = \frac{v_o}{v_i} = -g_m \left( \frac{R_C}{r_e + R_{\text{sig}}} \right)
\]

\[
G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \cdot \frac{v_o}{v_i} = \frac{r_e}{r_e + R_{\text{sig}}} g_m \left( \frac{R_C}{r_e + R_{\text{sig}}} \right)
\]

\[
= \alpha \frac{R_C}{r_e + R_{\text{sig}}}
\]

(4.93, 4.94)
Concludes:

1. The CB amplifier exhibits a very low input resistance \( r_e \).
2. The open-circuit voltage gain that is positive and equal in magnitude to that of the CE amplifier \( \left(g_m R_C\right) \).
3. The short-circuit current gain that is nearly unity \( \left(\alpha\right) \).
4. Like the CE amplifier, has a relatively high output resistance \( \left(R_C\right) \).
5. Has excellent high-frequency performance.

Because of its very low input resistance, the CB amplifier alone is not attractive as a voltage amplifier except in specialized application.
4.6.6 The CC Amplifier or Emitter Follower

The Need for Voltage Buffers

\[ R_{\text{sig}} = 100k\Omega \]

\[ \nu_{\text{sig}} = 200mV \]

\[ R_L = 1k\Omega \]

\[ R_{\text{sig}} = 100k\Omega \]

\[ \nu_{\text{sig}} = 200mV \]

\[ R_{\text{in}} = 100k\Omega \]

\[ A_o = 1 \]

\[ v_o \approx 99mV \]

\[ R_{\text{sig}} = 100k\Omega \]

\[ = 2mV \]

\[ R_{\text{sig}} = 100k\Omega \]
\[ i_b = (1 - \alpha)i_e = i_e/(1 + \beta) \]
\[ R_{\text{in}} = \frac{v_i}{i_b} = \frac{i_e (r_e + R_L)}{i_e /(1 + \beta)} = (1 + \beta)(r_e + R_L) \quad (4.95) \]

\[ A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + r_e} \quad (4.96), \quad A_{vo} \equiv \frac{v_o}{v_i} \bigg|_{R_L=\infty} \approx 1 \quad (4.97) \]

\[ R_o = r_e \quad (4.98) \]

**Overall Voltage Gain**

\[ \frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{(1 + \beta)(r_e + R_L)}{(1 + \beta)(r_e + R_L) + R_{\text{sig}}} \]

\[ G_v \equiv \frac{v_o}{v_{\text{sig}}} \cdot \frac{v_o}{v_i} = \frac{(1 + \beta)(r_e + R_L)}{(1 + \beta)(r_e + R_L) + R_{\text{sig}}} \cdot \frac{R_L}{R_L + r_e} \]

\[ = \frac{(1 + \beta)R_L}{(1 + \beta)r_e + (1 + \beta)R_L + R_{\text{sig}}} \approx 1 \quad (4.99) \]
\[ G_v = \frac{(1 + \beta)R_L}{(1 + \beta)r_e + (1 + \beta)R_L + R_{sig}} = \frac{R_L}{r_e + R_L + R_{sig}/(1 + \beta)} \]
Conclusions:

1. The CC amplifier exhibits a high input resistance, because of the $(1+\beta)$ multiplication factor.
2. Has a relatively large short-circuit current gain.
3. The open-circuit voltage gain that is smaller than but close to unity.
4. Has a relatively low output resistance.
Thévenin Representation of Emitter-Follower Output

\[ \frac{R_{\text{sig}}}{1 + \beta} \]

(a) \[ v_o = v_{\text{sig}} \]

\[ G_{\text{vo}} = 1 \]

(b) \[ R_{\text{out}} = r_e + \frac{R_{\text{sig}}}{1 + \beta} \]

(c) \[ R_{\text{in}} = (1 + \beta)(r_e + R_L) \]

(d)
Example 4.19: It is required to design an emitter follower to implement the buffer amplifier of Fig.4.54(c). Specify the required $I_E$ and the transistor $\beta_{\text{min}}$ must have. Determine the $\hat{v}_{\text{sig}}$ if $v_\pi$ is to limited to 5mV in order to obtain reasonably linear operation. with $v_{\text{sig}} = 200\text{mV}$, Determine the signal voltage $v_o$ if $R_L = 2k\Omega$ and to $0.5k\Omega$.

**Sol:**

From (4.45),

given $R_o = r_e = 10\Omega$

$$r_e = \frac{V_T}{I_E} \Rightarrow I_E = 2.5\text{mA} \quad \square$$

$$R_{\text{in}} = (1 + \beta)(r_e + R_L)$$

$$\Rightarrow 100 = (1 + \beta)(0.01 + 1)$$

$$\beta_{\text{min}} = 98 \quad \square$$
The overall voltage gain can be determined from

\[ G_v \equiv \frac{V_o}{V_{\text{sig}}} = \frac{R_L}{r_e + R_L + \left[ \frac{R_{\text{sig}}}{(1 + \beta)} \right]} \approx 0.5 \]

Thus when \( V_{\text{sig}} = 200\text{mV} \), the \( V_o = 200\text{mV} \cdot 0.5 = 100\text{mV} \)

\[ \Rightarrow V_\pi = \frac{V_o}{R_L} \times r_e = 1\text{mV} \], If \( \hat{V}_\pi = 5\text{mV} \), then \( \hat{V}_{\text{sig}} = 200\text{mV} \times 5 = 1\text{V} \]

Assume \( G_{V_o} = 1 \Rightarrow R_{\text{out}} = r_e + \frac{R_{\text{sig}}}{1 + \beta} = 0.01 + \frac{100}{101} = 1\text{k}\Omega \)

to obtain \( V_o = V_{\text{sig}} \frac{R_L}{R_L + R_{\text{out}}} \)

For \( R_L = 2\text{k}\Omega \) \( \Rightarrow V_o = 200\text{mV} \frac{2\text{k}\Omega}{2\text{k}\Omega + 1\text{k}\Omega} = 133.3\text{mV} \)

For \( R_L = 0.5\text{k}\Omega \) \( \Rightarrow V_o = 200\text{mV} \frac{0.5\text{k}\Omega}{0.5\text{k}\Omega + 1\text{k}\Omega} = 66.7\text{mV} \)
4.8 Discrete-Circuit BJT Amplifier

4.8.2 The common-Emitter (CE) Amplifier
\[ v_o = -g_m v_{\pi} \left( r_o // R_C // R_L \right) \]

\[ R_{ib} = r_{\pi} \]

\[ R_{in} = R_B // r_{\pi} \implies R_{in} \approx r_{\pi} \]

\[ R_{out} = \left( r_o // R_C \right) \]
4.8.3 The Common-Emitter (CE) Amplifier with an Emitter Resistance
\[ v_o = -\alpha i_e (R_C // R_L) \]

\[ R_{out} = R_C // r_o \approx R_C \]

\[ R_{ib} = (1 + \beta)(r_e + R_e) \]

\[ R_{in} = R_B // R_{ib} \]
4.8.4 The common-Base (CB) Amplifier

\[ v_o = -\alpha i_e (R_C \parallel R_L) \]

\[ R_{out} = R_C \parallel r_o \approx R_C \]

\[ R_{in} = \frac{v_i}{i_i} = r_e \]
4.8.5 The common-Collector (CC) Amplifier or Emitter Follower
\[ i_b = (1 - \alpha)i_e = \frac{i_e}{\beta + 1} \]
4.8.6 The Amplifier Frequency Response

(a) Capacitively coupled common-emitter amplifier.
The Three Frequency Bands

Low-frequency band
- Gain falls off due to the effects of $C_{C1}$, $C_{C2}$, and $C_{E}$

Midband
- All capacitances can be neglected

High-frequency band
- Gain falls off due to the effects of $C_{\pi}$ and $C_{\mu}$ of the BJT

$\frac{V_o}{V_{sig}}$ (dB)

$f_L$, $f_H$ (Hz) (log scale)

$20 \log |A_M|$ (dB)

3 dB
4.9 Transistor Breakdown and Temperature Effects

Figure 4.70 The BJT Common-base characteristic including the breakdown region.
Figure 4.71 The BJT Common-emitter characteristic including the breakdown region.
4.9.2 Dependence of $\beta$ on $I_C$ and Temperature

**Figure 4.72** Typical dependence of $\beta$ on $I_C$ and on temperature in a modern integrated-circuit $npn$ silicon transistor intended for operation around 1 mA.
Thanks For Your Attention!

Q & A